Carbon-Cycle Catastrophes: A Dynamical-Systems Perspective

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Earth's carbon cycles between photosynthesis, which converts carbon dioxide (CO_2) to organic carbon, and respiration—the metabolic processes that transform organic carbon to CO_2 [1,2]. As a result of human activities, atmospheric CO_2 levels have risen nearly 50% since pre-industrial times, and about two-thirds as much CO_2 has been absorbed by the oceans. There is much interest in predicting the climate's response to these changes. But a larger question seeks the consequences for the Earth system as a whole—not only for climate, but for the interactions of life and the environment. The geologic record shows that the five great mass extinction events of the last 540 million years are each associated with significant disruptions of the carbon cycle. So too are the periods of environmental change unassociated with mass extinction. Although the causes of all of these disruptions remain enigmatic, the disruptions themselves likely represent a qualitative change in the dynamics of the Earth system. Here I show how the theory of dynamical systems helps to understand these events.

The observational data sets the stage [3]. Figure 1A displays a geochemical time series containing two disruptions occurring about 54 million years ago. The sharp downward pulses can be interpreted as increases Δm in the mass m of inorganic carbon contained in the ocean. The amplitude and duration τ of the downswing can then be transformed to an estimate of $\Delta m/m$. Figure 1B shows the results obtained from 31 events during the last 540 million years. Roughly half of the events lie near the straight line. These *characteristic events* share a similar specific rate r_c in the relation $\Delta m/m = r_c\tau$. Although r_c is biogeochemically significant [3], it may also be dynamically significant, as it appears to separate four of the five mass extinction events from nearly all other disruptions.

Disruptions of the carbon cycle are usually interpreted as proportionate responses to perturbations, such as enhanced emissions of volcanic CO_2, extraordinary releases of methane, or changes in the rates at which organic carbon is sequestered in rock. But such a variety of stressors would seem unlikely to exhibit a common specific rate. Characteristic events may instead reflect the intrinsic dynamics of the carbon cycle.

A simple model of the marine carbon cycle illustrates how this could work [4]. We consider the upper ocean to be a well-mixed chemical reactor open to an incoming flux j_i of dissolved calcium carbonate from rivers and an outgoing flux of maximum strength bj_i to sediments. Within the ocean, dissolved inorganic carbon takes several forms; it suffices to consider only two. We track of the concentration of total dissolved inorganic carbon (carbonate and bicarbonate ions in addition to CO_2), denoted by w, and the concentration of carbonate ions (CO_3^{2-}), denoted by c, and allow for external input of CO_2 at rate νj_i.

One important feedback concerns the outgoing flux. Above a critical concentration c_p, carbonate minerals are preserved in sediments; below c_p, they dissolve. Averaged over the oceans, the transition is smooth rather than sharp; the outflux is therefore taken to be proportional to a sigmoidal function $s(c, c_p)$ that grows from zero to one as c increases to values well above c_p. Another feedback concerns the interaction of the carbonate system with planktonic organisms. When CO_2 is added to seawater, some of it combines with carbonate ions to form bicarbonate ions. The carbonate ion concentration c then becomes smaller. If c becomes sufficiently small—less than a concentration c_x—planktonic organisms that make shells of calcium carbonate fare poorly. Their shells normally provide "ballast" that causes these organisms to sink to the deep sea along with detrital organic carbon. But if fewer calcifying organisms exist in the upper ocean, less organic carbon sinks to the seafloor and more is respired to CO_2 in the shallow ocean. Upper ocean CO_2 levels then increase even more. We express this process by a constant θ times a sigmoidal function $\bar{s}(c, c_x)$, where $\bar{s} = 1 - s$. Finally, we assume that the concentration of total dissolved inorganic carbon tends to relax toward a concentration w_0 at a timescale τ_w. Today, $\tau_w \sim 10^4$ yr. These considerations lead to the *carbon cycle model:*

$$\dot{c}/f(c) = j_i[1 - bs(c, c_p) - \theta\bar{s}(c, c_x) - \nu] + (w - w_0)/\tau_w \quad (1)$$

$$\dot{w} = j_i[1 - bs(c, c_p) + \theta\bar{s}(c, c_x) + \nu] - (w - w_0)/\tau_w. \quad (2)$$

The function $f(c)$ approximates how the addition or removal of chemical species is "buffered" by the carbonate system.

Depending on parameters, the model exhibits either a stable fixed point, a stable limit cycle, or both. Here I focus on the response to perturbation of a stable fixed point when it is near the bistable region. We imagine that the system is initially prepared with the CO_2 injection parameter $\nu = 0$, but at time $t = 0$ we set $\nu = \nu_0 > 0$ for all times $t \geq 0$. Figure 2A-B shows that the system spirals toward a new steady state. However, when ν_0 is larger than a threshold ν_c, the system undergoes a large excitation before it approaches the new fixed point (Figure 2C-D). The model is therefore *excitable*, similar to models of action potentials in a neuron [5]. Here, the excitation corresponds to transient ocean acidification (since the addition of CO_2 not only reduces c but also pH.)

Because the size and timescale of excitations are properties of the system rather than its perturbation, an excitable carbon cycle could explain several features of the data in Figure 1B [4]. Characteristic events—those near the straight line—would represent near-threshold excitations. The mass extinction events above the line would be associated with perturbations that significantly exceed the threshold. Events lying below the line may simply represent slow quasistatic change.

In the real carbon cycle, the initiation of an excitation by CO_2 injection would likely last only for a finite time t_i, e.g., $\nu = \nu_0$ for $0 \leq t \leq t_i$. In the carbon cycle model, the excitation threshold ν_c is independent of t_i when t_i exceeds approximately one damping time τ_w. But if $t_i \ll \tau_w$, perturbations can be damped before they lead to excitation. In this case, the threshold depends on the total mass injected and $\nu_c \propto t_i^{-1}$. Figure 3A-B illustrates how this works. Similar phenomena exist in other excitable systems [5, 6].

The flux of CO_2 entering today's oceans is nearly two orders of magnitude greater than the growth rate of characteristic events, which may be taken to be a rough estimate of the threshold's upper bound under a step-like injection. Yet the centennial timescale of the modern perturbation is about two orders of magnitude shorter than today's damping timescale. The shorter injection timescale cancels out the stronger injection rate, and, rather than greatly exceeding the hypothesized upper bound, the modern influx is "merely" near it.

To understand what this might mean, consider the behavior of the carbon cycle preceding the end-Cretaceous extinction (and the demise of dinosaurs). Although the extinction is widely attributed to a bolide impact, modern geochronological methods reveal a roughly 10^4-yr pulse of massive volcanism tens of thousands of years before the impact [7]. The resulting CO_2 injection occurred at an estimated peak rate that is approximately 1% of the maximum projected 21st-century mean flux to the oceans [4]. Both the modern and end-Cretaceous CO_2 fluxes therefore lie near the threshold's upper bound, as shown in Figure 3C. Massive volcanism associated with the end-Permian and end-Triassic extinction events may be similarly pulsed.

The upshot is twofold. First, modest perturbations of the carbon cycle beyond a threshold may have led to significant disruptions of the ancient Earth system, possibly including mass extinction. Second, today's strong perturbation appears to be near an equivalent threshold. Are we therefore headed for a sixth extinction? The dynamical system of equations (1) and (2) is a toy model; among other limitations, its assumption of a well-mixed ocean neglects possible feedbacks on timescales less than about 10^3 yr. Yet the hypothesis of excitability is reasonable and the t_i^{-1} scaling law provides something new and important: a way to rescale the past to the present. Analysis of alternative models and acquisition of more data will help test these ideas. The risk of catastrophe makes fundamental progress imperative.

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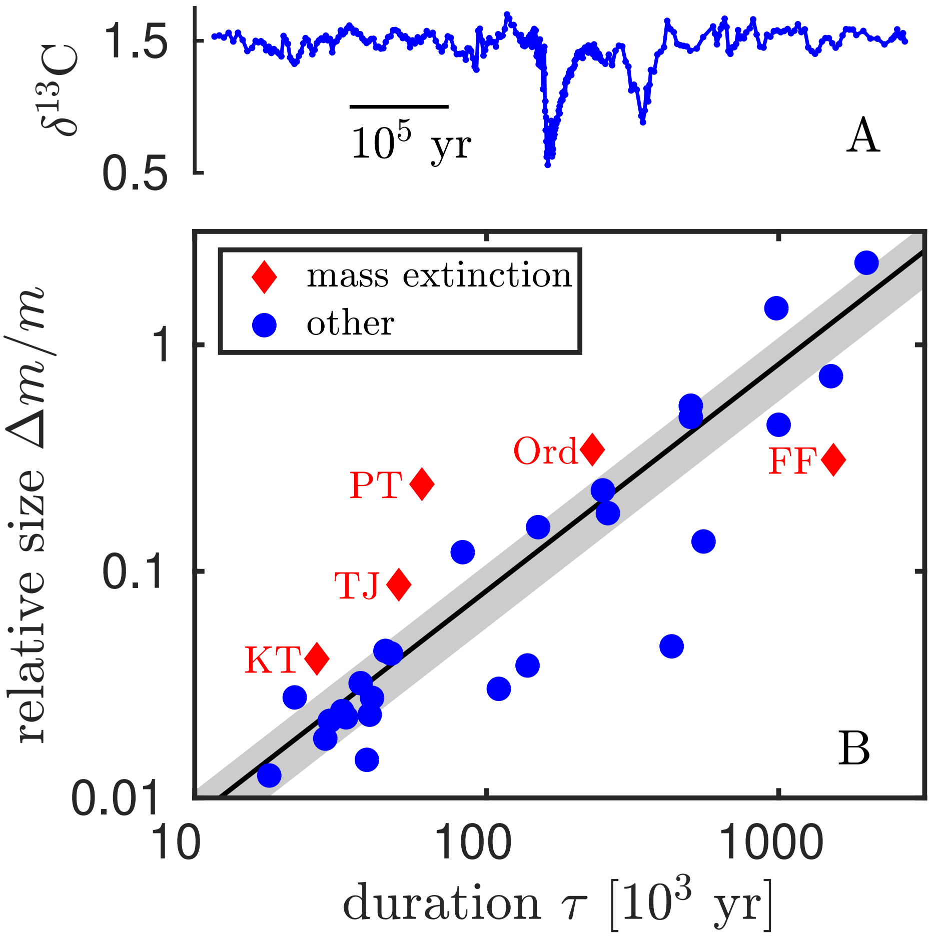

Figure 1: Disruptions of the carbon cycle [3]. (A) Time series of the isotopic composition of carbonate carbon (δ^{13}C, the relative enrichment of ^{12}C compared to ^{13}C, expressed as "per mil") during the Eocene period, about 54 million years ago. The two abrupt downswings may be transformed into increases Δm in the mass m of dissolved inorganic carbon in the oceans. (B) The relative size and duration of 31 global disruptions in the last 540 million years. The duration τ is the time over which m grows. The labeled events are associated with the end-Cretaceous (KT), end-Triassic (TJ), end-Permian (PT), end-Ordovician (Ord), and Frasnian-Famennian (FF) mass extinctions.

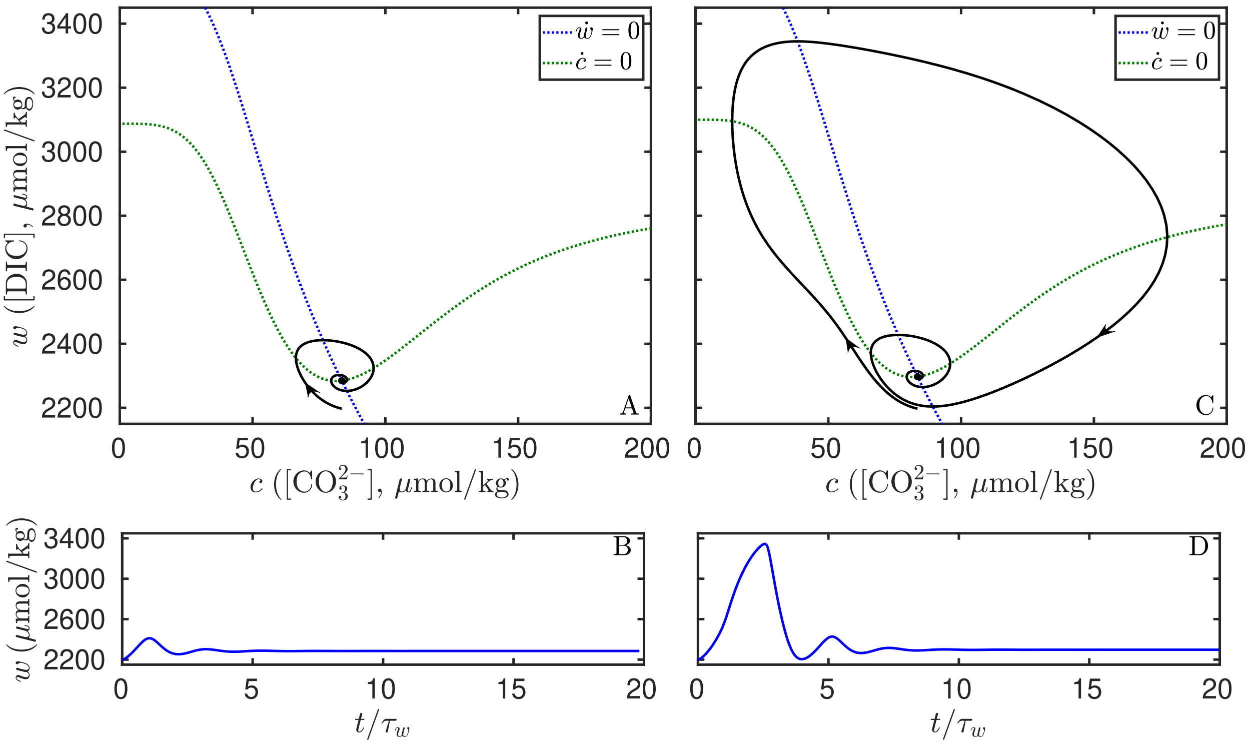

Figure 2: Phase plane and time series representations of perturbations of the carbon cycle model below (A,B) and above (C,D) the excitation threshold [4].

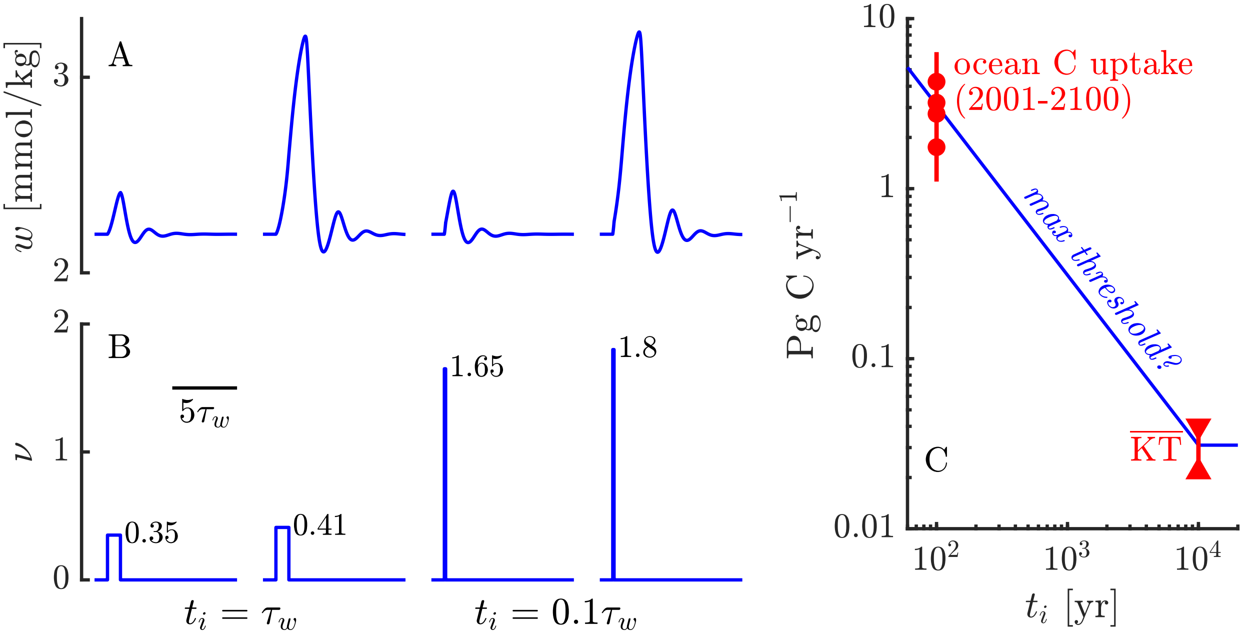

Figure 3: The excitation threshold ν_c depends on the duration of forcing. (A) The response $w(t)$ to (B) injection of CO_2 at different dimensionless rates ν over different durations t_i. (C) Comparison of perturbations of the modern and end-Cretaceous [7] carbon cycles to a hypothesis for the upper bound of the excitation threshold. Four projections of the modern perturbation are indicated [4]. The end-Cretaceous case represents the upper half of the confidence interval for CO_2 emitted by massive volcanism tens of thousands of years before the end-Cretaceous extinction [7]. Because the excitation threshold scales like t_i^{-1}, both the modern and ancient perturbations are equivalently near it.